

SOIL SLOPES UNDER COMBINED HORIZONTAL AND VERTICAL SEISMIC ACCELERATIONS

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SUMMARY

Conventional methods of designing earth structures are based on pseudo-static stability analysis employing a horizontal seismic coefficient. This paper discusses the stability and permanent displacement of a slope subject to combined horizontal and vertical accelerations. A log-spiral failure mechanism is used. It is shown that seismic force has a significant effect on stability and permanent displacement of slopes. The parametric study reveals that vertical acceleration may play an important role on stability and permanent displacement if the corresponding horizontal acceleration is large. © 1997 John Wiley & Sons Ltd.

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INTRODUCTION

Seismic designs of earth structures, such as slopes, retaining walls, embankments and dams, are conducted using pseudo-static approach. The Mononobe–Okabe approach,^{1,2} used in determining lateral earth pressure, is among the most widely used pseudo-static procedures. The seismic inertia force, whose magnitude and direction vary throughout the duration of excitation, is regarded as pseudo-static and represented using horizontal and vertical seismic coefficients. These coefficients are expressed as a percentage of the earth gravity. Conventional design is, however, conducted using primarily a horizontal seismic coefficient.

Seismic coefficient is determined from experience, typically based on the maximum horizontal acceleration (MHA) or peak ground acceleration (PGA) of a design earthquake. Seed and Martin³ suggested this value to be between 0.05 and 0.2 for earth dam. The seismic risk maps of AASHTO⁴ and USGS,⁵ for instance, provide useful information on seismic coefficient based on probability of earthquake occurrence. It may, however, be unconservative to adopt bedrock acceleration from these maps for design since subsoil conditions may lead to amplification of acceleration. For some earth structures, amplification of acceleration in the structures itself could be a concern and specific response analysis may be warranted.

Past earthquake records show that peak value of vertical acceleration is in the range of half to two-thirds that of horizontal acceleration.^{6,7} For a structure designed to resist only a horizontal acceleration, and if it is

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subjected to an earthquake having a maximum horizontal acceleration smaller than this design value, the inherent safety margin would compensate for the vertical inertia force. Recent earthquakes, such as 1994 Hokkaido Toho-Oki Earthquake⁸ ($M = 7.9$), 1989 Loma Prieta Earthquake⁹ ($M = 7.1$), 1994 Northridge Earthquake¹⁰ ($M = 6.7$), and 1995 Hanshin Earthquake¹¹ ($M = 7.2$), recorded a significantly large ratio of peak vertical to horizontal acceleration. For some of these records, the peak vertical acceleration was as large as that of corresponding horizontal acceleration.

Possible effect of vertical acceleration on the stability of earth structures has not been discussed widely although it had been brought to attention as early as 1920s following devastating Kanto Earthquake in Japan. Mononobe¹ showed that a combination of horizontal and vertical accelerations led to severe damage of earth retaining structures. Chopra,¹² through finite element analysis, showed that vertical acceleration affects seismic response of earth dam. Vertical acceleration has also been an issue of debate following recent Northridge and Hanshin Earthquakes, but its possible effects are yet to be investigated.

The objective of this paper was, thus, to examine the stability of soil slopes under a combination of horizontal and vertical accelerations, and subsequent permanent displacement using real earthquake acceleration records. A log-spiral mechanism was formulated and used in this study.

SEISMATIC STABILITY

A homogeneous slope of height H and inclination i is considered. The unit weight of soil is expressed as γ . It is assumed that soil obeys well-known Coulomb failure criterion, expressed by two strength parameters: cohesion and angle of internal friction (c, ϕ). In design, cohesion and frictional components of the strength are reduced by a designated factor of safety, F_s . They are usually expressed as dimensionless parameters, N_m and ψ_m :

$$N_m = \frac{c}{F_s \gamma H} \quad (1)$$

$$\psi_m = \frac{\tan \phi}{F_s} \quad (2)$$

Design chart, such as that proposed by Taylor,¹³ gives the relationships between N_m and ψ_m for different slope angles. These charts are widely used to assess the static stability of slopes.

Pseudo-static seismic stability analysis using a planar mechanism has been addressed by Seed and Goodman,¹⁴ Sarma,¹⁵ among others, based on a resultant earthquake acceleration acting at a prescribed failure plane. Sarma showed that both the factor of safety and permanent displacement are not sensitive to the direction of resultant acceleration.

Pseudo-static seismic stability using a log-spiral failure mechanism has been considered by Prater,¹⁶ Koppula,¹⁷ Chen and Sawada,¹⁸ Leshchinsky and San,¹⁹ among others. All these studies considered only a horizontal component of seismic acceleration. The studies of Prater and Koppula are limited to cohesive soils. It has to be noted that the study by Chen and Sawada was based upon an upper-bound limit analysis using the energy theorem. Leshchinsky and San¹⁹ introduced a horizontal seismic coefficient to the variational approach of Baker and Garber²⁰. This formulation is based on limit equilibrium approach but not the law of plasticity. It determines the minimum factor of safety for a log-spiral mechanism in which all static equilibrium equations are satisfied without *a priori* assumptions.

The rotational log-spiral failure surface is used in this study. The earthquake inertia force is represented using horizontal and vertical seismic coefficients: k_h and k_v . Note that positive k_v is considered to act downwards. The effect of pore water pressure is not included, thus the formulations are valid for free-draining soils. The governing differential equation and solution of normal stress (S) distribution along the log-spiral failure surface, incorporating horizontal and vertical seismic coefficients, are given in Appendix II.

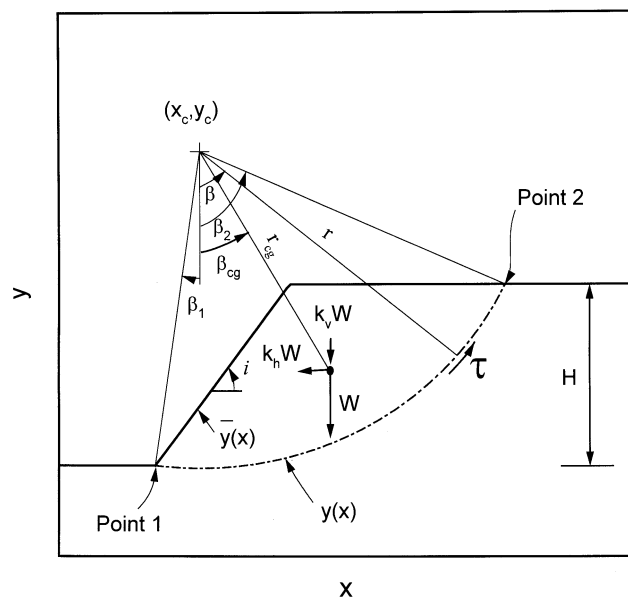


Figure 1. Seismic stability analysis with log-spiral failure surface

In the log-spiral analysis, the following parametric equations are adopted with the polar co-ordinates (R, β) :

$$X = X_c + R \sin \beta \quad (3)$$

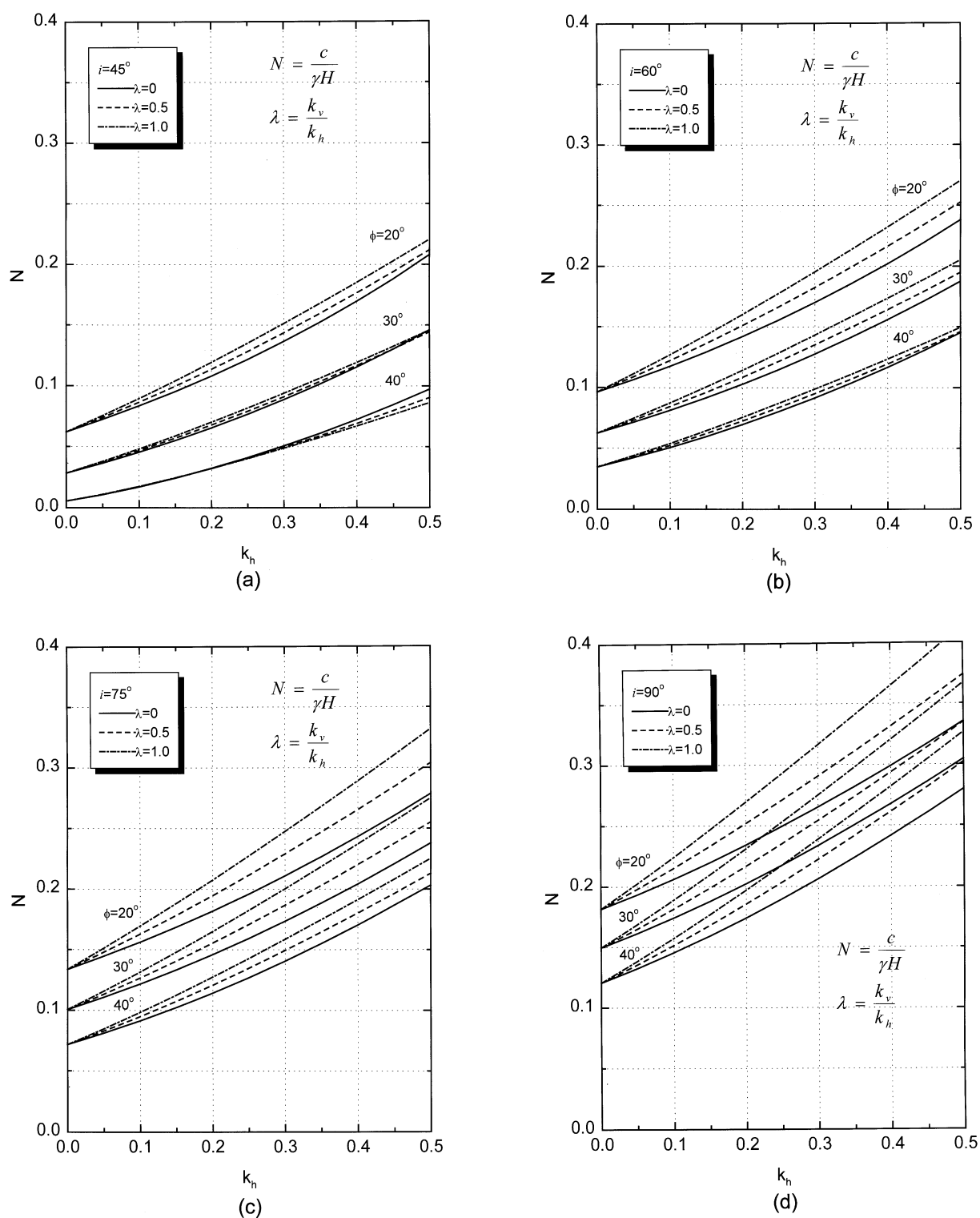
$$Y = Y_c - R \cos \beta \quad (4)$$

$$R = A \exp(-\beta \psi_m) \quad (5)$$

where $(X = x/H, Y = y/H)$ and (X_c, Y_c) are the normalized co-ordinates of failure surface and the pole of log-spiral, respectively. β , A , and R are the angle of inclination of the point of interest, log-spiral constant, and normalized log-spiral arm length, respectively (Figure 1). Consequently, for a soil mass bounded by the slope surface $\bar{Y}(X)$ and slip surface $Y(X)$, the moment equilibrium equation about the pole of the log-spiral is written as

$$\begin{aligned} N_m \int_{\beta_1}^{\beta_2} [(Y - Y_c) - (X - X_c) Y'] (\cos \beta - \psi_m \sin \beta) \exp(-\beta \psi_m) d\beta \\ + (1 + k_v) \int_{\beta_1}^{\beta_2} (\bar{Y} - Y)(X - X_c) (\cos \beta - \psi_m \sin \beta) \exp(-\beta \psi_m) d\beta \\ + \frac{k_h}{2} \int_{\beta_1}^{\beta_2} (\bar{Y} - Y)(Y + \bar{Y} - 2Y_c) (\cos \beta - \psi_m \sin \beta) \exp(-\beta \psi_m) d\beta = 0 \end{aligned} \quad (6)$$

where β_1 and β_2 are angles bounded by the failure surface (see Figure 1), and $Y' = dY/dX$ is the tangent of failure surface. In equation (6), the first, second and third terms represent contributions from the shear strength, deadweight of soil mass with vertical seismic force, and horizontal seismic force, respectively. It is

Figure 2. Relationships between soil strength and seismic coefficients: (a) $i = 45^\circ$, (b) $i = 60^\circ$, (c) $i = 75^\circ$, (d) $i = 90^\circ$

solved to yield the most critical log-spiral as defined by X_c , Y_c and A . Closed-form solution to equation (6) may be sought, but in this study, it is obtained by dividing potential failure soil mass into slices, and integrated numerically, similar to the procedures presented under section "Computational Procedure" of Leshchinsky and San.¹⁹

Considering a factor of safety of unity, N_m and ψ_m are expressed as N and ψ , respectively. Figures 2 show N for slopes of inclination i (45, 60, 75, 90°) and ϕ (20, 30, 40°) under different values of seismic coefficient. The ratio of vertical to horizontal seismic coefficients is expressed as λ . Only results for $+\lambda$, which is more critical than $-\lambda$ in most steep slopes, are shown. It can be seen that N increases for an increased slope inclination. For example, at $\phi = 30^\circ$, $k_h = 0.2$ and $\lambda = 0$, N is about 3 times or more in a vertical slope when compared to slope of $i = 45^\circ$. For $i = 45^\circ$, the effect of λ is negligibly small, but as i increases to 75 and 90°, the effects becomes significantly large.

PERMANENT DISPLACEMENT

The factor of safety is used to design or assess slope stability in a pseudo-static approach. While this is a convenient design procedure, physical interpretation of this factor of safety is not apparent under seismic loading conditions since the time response of a slope is not considered. On the contrary, sliding block theory proposed by Whitman²¹ and Newmark²² is considered a more appropriate tool to assess slope performance. In the sliding block theory, yield acceleration of the slope, i.e. when factor of safety equal to unity, is determined. The relative acceleration of the block is then double-integrated to obtain permanent displacement. In this procedure, reverse yield acceleration is considered sufficiently large such that effects of reverse acceleration may be neglected in calculation. Such an assumption may give conservative results.

Permanent displacement of slopes has been examined to a certain extent by Goodman and Seed,²³ Sarma,²⁴ Chang *et al.*,²⁵ Sawada *et al.*,²⁶ Ling and Leshchinsky,²⁷ among others. Goodman and Seed²³ considered a planar failure surface of cohesionless soil. Sarma²⁴ used a circular mechanism while Sawada *et al.*²⁶ and Ling and Leshchinsky²⁷ used a log-spiral mechanism. Lin and Whitman²⁸ utilized a probabilistic approach to estimate better the permanent displacement by considering uncertainties of earthquake characteristics. Franklin and Chang,²⁹ and Makdisi and Seed³⁰ compiled design charts from a large number of earthquake records for estimating possible permanent displacement. Ambraseys and Srbulov³¹ also made significant contribution to estimate better the permanent displacement using simple parameters, such as earthquake magnitude, distance from the epicenter, and the critical acceleration ratio. While different failure mechanisms were examined by these investigators, the effect of vertical acceleration has not been addressed adequately in the context of permanent displacement.

In a log-spiral mechanism, yield seismic coefficient for a given slope geometry and soil properties may be calculated iteratively from equation (6) at $F_s = 1.0$, by setting $k_h = k_{hy}$. For convenience of computation, k_v may be expressed as λk_{hy} to avoid using separate set of horizontal and vertical acceleration records, i.e., the vertical acceleration is considered to be in phase with the horizontal acceleration. Although a constant value of k_{hy} is normally used in analysis, ϕ or k_{hy} may also be considered to vary with time under seismic excitation.

Limit equilibrium analysis assumes simultaneous mobilization of shear stress along the failure surface. By assuming small rotation, possible lost of energy by internal shear in the soil mass as well as the change of critical acceleration due to the change in geometry may be neglected. From the equation of angular motion, rotational acceleration of a 'rigid' failure soil mass is obtained as

$$\ddot{\theta} = \frac{\cos \beta_{cg}}{r_{cg}} (k_h - k_{hy})g \quad (7)$$

where r_{cg} is the distance from the centre of gyration to the pole of log-spiral, and β_{cg} is its inclination measured from the vertical. Rotation relative to the stable base is obtained by double integrating equation

(7), from which horizontal and vertical permanent displacements, at any point along the log-spiral surface, are obtained.

$$\delta_x/H = R \cos \beta, \quad \delta_y/H = R \sin \beta \quad (8)$$

Ling and Leshchinsky²⁷ presented relationships between permanent rotation/displacement and seismic acceleration for slopes of different inclinations and properties. However, the results were limited to harmonic sine wave using a prescribed frequency and duration of excitation. This is obviously in contradiction to a real earthquake acceleration which is random in nature. There also exists dilemma as to the selection of a representative value of earthquake frequency and duration of excitation in design. In other words, the correlation between harmonic and real earthquake motions is not apparent for practical use.

In this study, real earthquake records are used. The records were scaled to different peak values, k_{ho} , to calculate for rotation and permanent displacement. In an actual design, it is advised that past earthquake records from the site or from sites with comparable ground conditions to be used. The procedure of numerical integration of these random earthquake records follows that presented in Reference 27 by assuming a linear variation of acceleration in small time intervals t and $t + \Delta t$.

Figure 3(a) shows the response of a 60°-slope, $\phi = 30^\circ$, $k_{ho} = 0.2$, $k_{hy} = 0.1$, $\lambda = 0$, calculated for the Imperial Valley Records of the 1940 El Centro Earthquake. There are a few spikes throughout duration of excitation where yield acceleration is exceeded, leading to permanent rotation and displacement of slope along the failure surface relative to stable base. The permanent rotation is calculated to be about 0.1°. Figure 3(b) shows the relationships between seismic coefficient and permanent rotation/toe horizontal

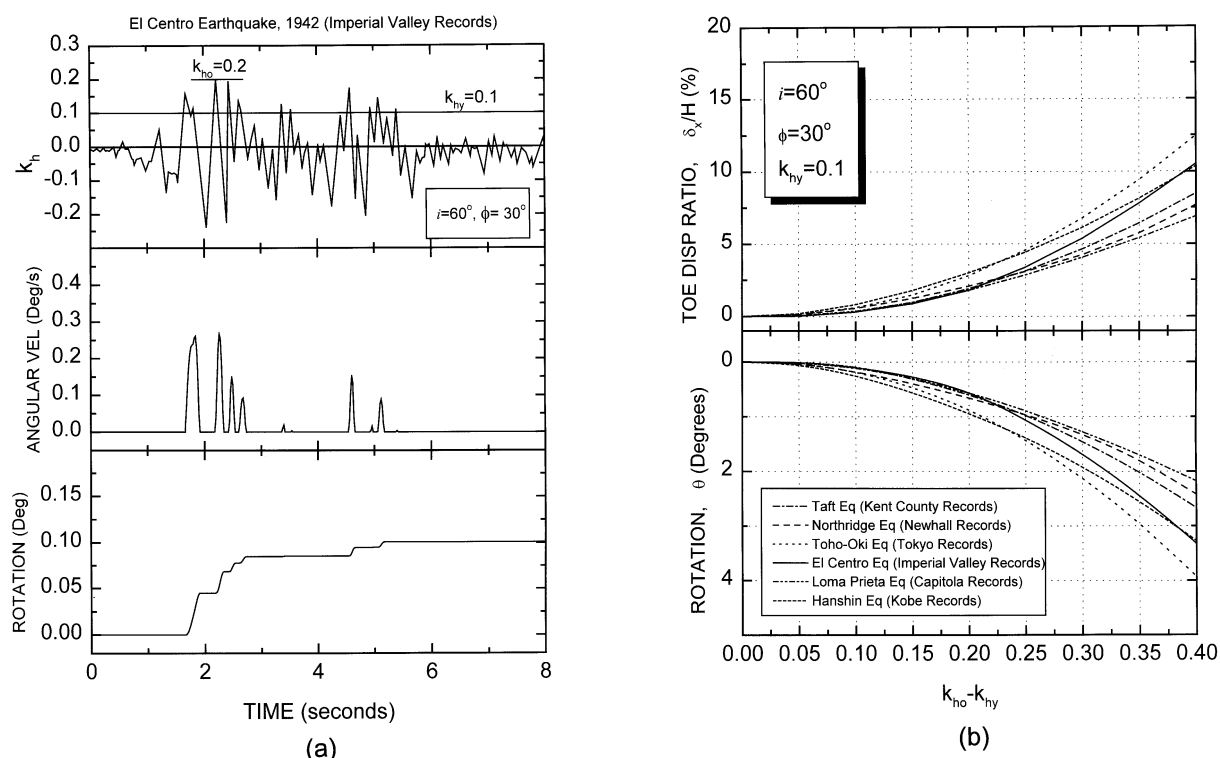


Figure 3. (a) Seismic response of a typical slope. (b) Permanent rotation and horizontal displacement at the toe under different earthquake records

displacement ratio for this 60° slope, calculated from six representative earthquake records obtained from El Centro Earthquake, Taft Earthquake, Loma Prieta Earthquake, Northridge Earthquake, Toho-Oki Earthquake, and Hanshin Earthquake. It is seen that there is a slight difference in the calculated permanent horizontal displacement for the range of seismic coefficients used. In the parametric studies followed, permanent displacement was calculated using Imperial Valley records of the El Centro Earthquake as given in Chopra.³²

PARAMETRIC STUDIES

The relationships between seismic coefficient and permanent displacement of slopes under a log-spiral mechanism are shown in Figures 4 ($\phi = 30^\circ$, $i = 45^\circ, 60^\circ, 75^\circ$ and 90°). The results show that displacement increases with an increase in the acceleration and slope angle. The permanent displacement calculated from real earthquake records (Imperial Valley records) is considerably smaller than that based on harmonic wave, say at a frequency of 1 Hz and duration of excitation of 10 s (see Reference 27), which may be considered equivalent to real earthquake records.

The curves presented in Figures 4 would be useful for a permanent displacement design of man-made structures, such as reinforced soil structures and waste containment systems. Some aspects of this design are addressed in References 33 and 34. In reinforced soil structure design, for example, N presented in Figures 2 is related to geosynthetic tensile reinforcement force required to stabilize a slope, and the location of failure surface determines the required geosynthetic length considering rotational mode of failure.

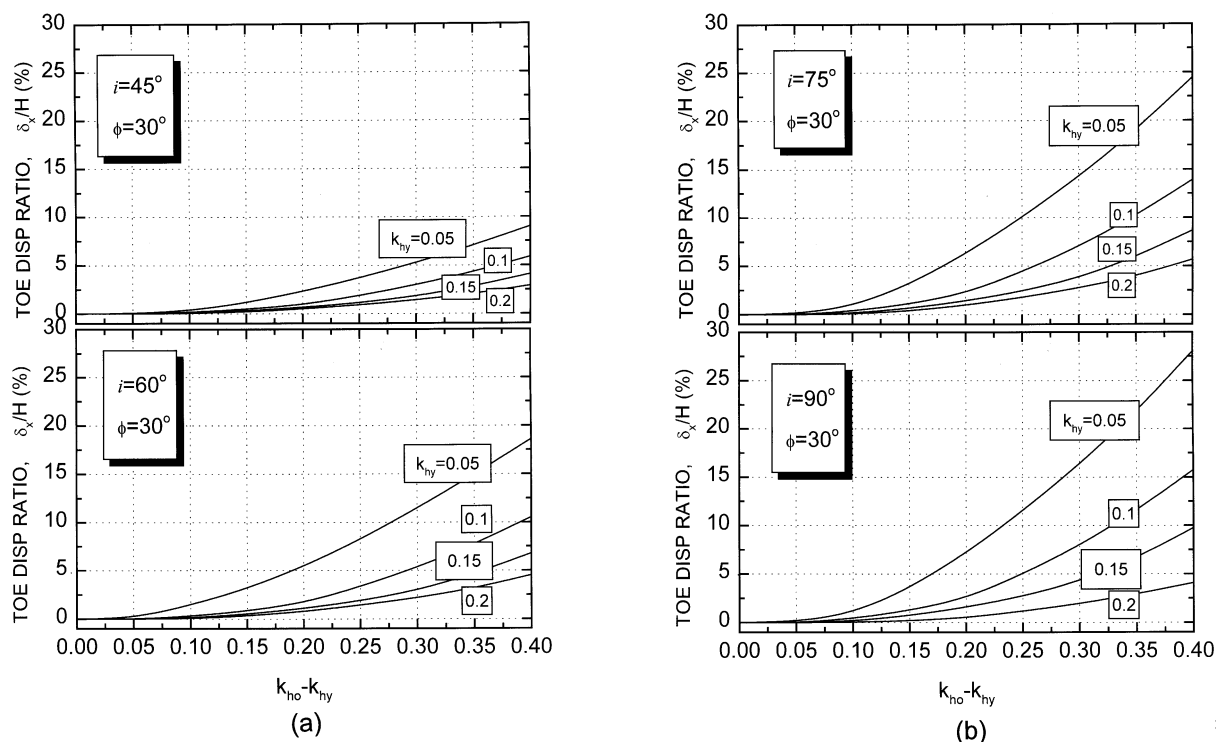
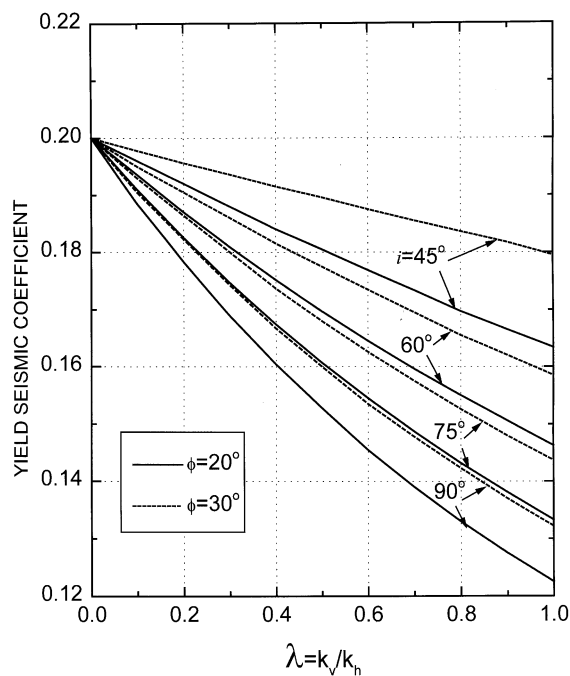
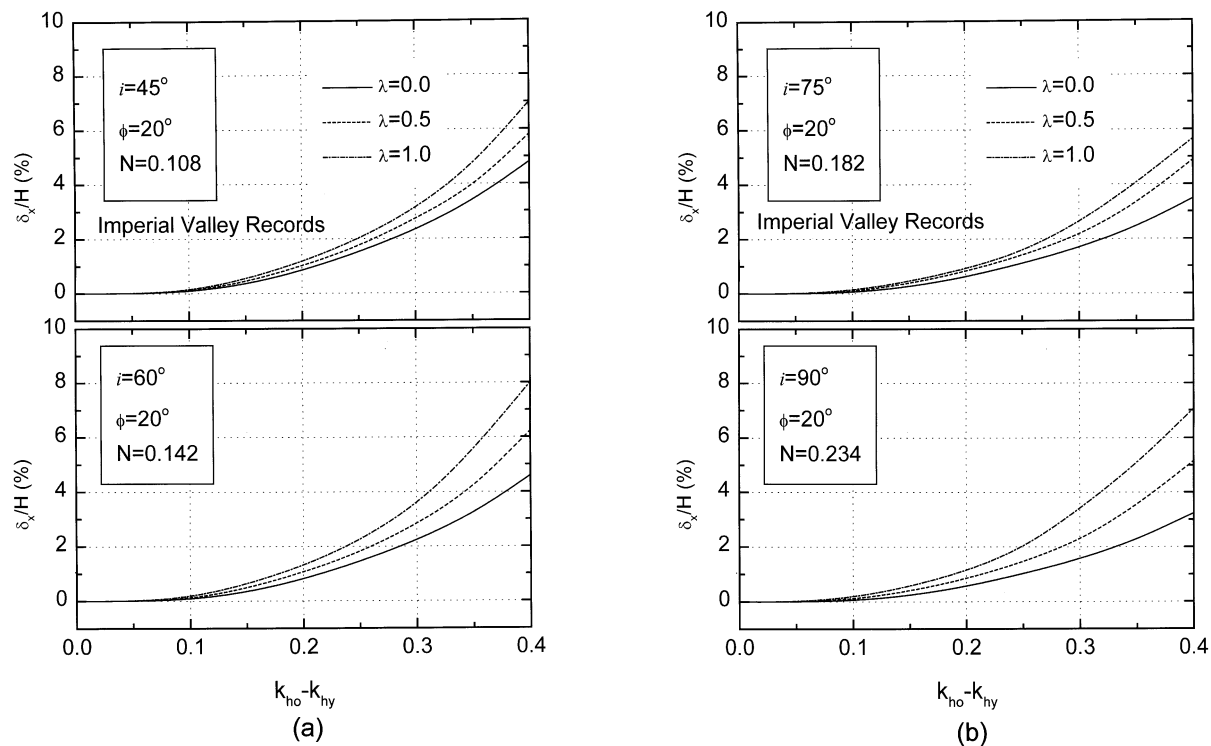


Figure 4. Relationships between horizontal permanent displacement at the toe and seismic coefficients, $\phi = 30^\circ$: (a) $i = 45$ and 60° , (b) $i = 75$ and 90°

Figure 5. Effects of vertical acceleration on yield seismic coefficient, $\phi = 20$ and 30° Figure 6. Effects of vertical acceleration on horizontal displacement at the toe, $\phi = 20^\circ$: (a) $i = 45$ and 60° , (b) $i = 75$ and 90°

The effects of vertical acceleration on permanent displacement are investigated for slopes designed with different values of vertical acceleration. Note that vertical acceleration is expressed by a ratio $\lambda = k_v/k_h$ in computation assuming k_v to be in phase with k_h . Figure 5 shows that for slopes designed with $k_{hy} = 0.2$ (without considering vertical acceleration), the yield seismic coefficient reduces if vertical acceleration actually exists. For $\phi = 20$ and 30° , the reduction is significant for vertical slopes when compared to 45° slopes.

Figures 6 show the calculated permanent displacement for slopes having $\phi = 20^\circ$ and λ equal to 0.05 and 1.0. Note that all slopes shown in Figures 6 have different value of N at $F_s = 1.0$. The effect of acceleration on slope displacement is seen for all slope inclinations. The results also show that effects of vertical acceleration on high slopes can be of practical significance if the corresponding horizontal acceleration is large.

CONCLUSIONS

Seismic stability and permanent displacement of slope were examined using a log-spiral mechanism. A vertical component of seismic inertia force, which is typically neglected in current design, was incorporated into the analysis.

This study showed that vertical acceleration has a significant effect on the calculated factor of safety and yield acceleration of steep slopes. The calculated permanent displacement was affected by vertical acceleration when corresponding horizontal acceleration is large. A vertical acceleration acting towards gravity may be included for design of slopes that are located adjacent to important facilities, with λ equal to, for example 0.5, as proposed for the Japanese atomic power facilities.³⁵ The study revealed that usage of real earthquake records yielded considerably smaller permanent displacement when compared to that calculated assuming a harmonic motion. This is mainly due to the dilemma of specifying representative value of frequency and duration of excitation in a harmonic motion.

It has to be noted that for a more precise seismic evaluation of slopes, sophisticated dynamic response analysis is warranted in addition to simplistic pseudo-static analysis.

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APPENDIX I

Notations

A, B	log-spiral constant, constant of integration
c	cohesion
F_s	factor of safety
H	height of slope
i	slope inclination
k_h, k_v	horizontal and vertical seismic coefficients
k_{ho}, k_{hy}	peak and yield seismic coefficient
N_m, N	normalized cohesion or stability number
R	normalized log-spiral arm length
r_{cg}	arm length from log-spiral pole to centre of gyration of failure soil mass
S	normalized shear strength

$t, \Delta t$	time, time increment
W	dead weight of failure soil mass
X, Y	normalized x- and y-co-ordinates
X_c, Y_c	normalized log-spiral pole co-ordinates
\bar{Y}, Y'	elevation and tangent of failure surface
β_1, β_2	angle bounded by failure surface
β, β_{cg}	angle of inclination of any point, centre of gyration, measured from the pole
δ_x, δ_y	horizontal and vertical displacements
ϕ	internal friction angle of soil
γ	unit weight of soil
λ	ratio of vertical to horizontal seismic coefficients
$\theta, \dot{\theta}, \ddot{\theta}$	rotation, angular velocity and acceleration of failure soil mass
τ	shear stress
ψ_m, ψ	normalized frictional strength

APPENDIX II

$$\frac{dS}{d\beta} - 2S\psi_m + R\{(1 + k_v)\sin\beta + k_h\cos\beta\} - 2N_m = 0 \quad (9)$$

$$S(\psi_m \neq 0) = \frac{A}{1 + 9\psi_m^2} [\{(1 + k_v) + 3\psi_m k_h\} \cos\beta + \{3(1 + k_v)\psi_m - k_h\} \sin\beta +] \exp(-\beta\psi_m) \\ - N_m \frac{1 - \exp(2\beta\psi_m)}{\psi_m} + B \exp(2\beta\psi_m) \quad (10)$$

$$S(\psi_m = 0) = A\{(1 + k_v)\cos\beta - k_h\sin\beta\} + 2N_m\beta + B \quad (11)$$

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